

Gauge-free Coleman-Weinberg Potential

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Abstract

The gauge-dependence of the one loop Coleman-Weinberg effective potential in scalar electrodynamics is resolved using a gauge-free approach not requiring any gauge-fixing of quantum fluctuations of the photon degrees of freedom. This leads to a unique dynamical ratio at one loop of the Higgs mass to the photon mass. We compare our approach and results with those obtained in geometric framework of DeWitt and Vilkovisky, which maintains invariance under field redefinitions as well as invariance under background gauge transformations, but *requires*, in contrast to our approach, gauge fixing of *fluctuating* photon fields. We also discuss possible modifications of the Coleman-Weinberg potential if we adapt the DeWitt-Vilkovisky method to our gauge-free approach for scalar QED.

1 Introduction

The Coleman-Weinberg effective potential has been a very important tool to study the vacuum structure and radiative Higgs mechanism for mass generation in gauge field theory. Since the seminal paper by Coleman and Weinberg [1], a lot of effort have been made to calculate the effective potential in a systematic manner. One of the most significant works in this context was made by Jackiw [2] where the one loop effective action is computed using loop-expansion in a purely functional integral scheme. It is also shown in this work that the one loop effective potential is gauge *dependent*, and in fact could be gauged away within a class of gauges by appropriate choice of a gauge parameter. The issue of gauge dependence of the effective action has been extensively analyzed in the past [see [3] and the references therein]. It has also been pointed out that the effective action is not only gauge dependent but also depends upon field reparameterization [4]. A new approach for computing the effective action was introduced by DeWitt and Vilkovisky [4, 5], where the effective action was explicitly shown to be reparameterization independent. Further, this DeWitt-Vilkovisky scheme used a background field method and the effective action is explicitly background gauge invariant, even though it requires a choice of gauge to integrate the fluctuating part of the gauge fields.

Our aim in this paper is to recalculate the effective potential for scalar QED at one loop, using an approach which we call *gauge-free*. In this framework, quantum fluctuating dynamical

variables are *manifestly inert* under (abelian) gauge transformations [6], [7]. In contrast to the usual treatment of functional quantization of gauge theories involving the Faddeev-Popov ansatz, here we propose a reformulation of electrodynamics in terms of a *physical* vector potential entirely free of gauge ambiguities right from the outset [6], **and which is spacetime divergenceless** : $\partial \cdot \mathbf{A}_{\mathcal{P}} = 0$. It is important to note that this last property of the vector potential is *not* the Lorentz gauge condition but a *physical restriction* on a physical vector potential. It is merely a restatement of the fact that in the standard formulation of pure electrodynamics, gauge transformations act only on the unphysical degrees of freedom of the gauge potential, with the physical, gauge invariant part of the gauge potential being divergenceless by definition, not as a matter of choice. The gauge free approach, by virtue of being based on a physical, divergenceless vector potential, evades the entire issue of gauge redundancy. Quantizing the theory with this prescription leads to a propagator that is gauge invariant by construction, in contrast to the standard photon propagator.

The charged matter fields can be coupled with this *physical* photon field in a gauge-free fashion if we rewrite the fields in polar representation, so as to ‘separate’ charge and spin degrees of freedom. The modulus of the matter fields carries the spin (scalar) degrees of freedom and the phase part carries only the charge of it. This separation of spin and charge actually enables us to represent the theory in terms of manifestly gauge-inert variables.

We would like to mention here that the radial decomposition of charged matter fields sometimes been referred to as imposing ‘unitarity gauge’, in the literature [8, 9]. We do not agree with this notion because “unitarity gauge” is not really a choice of gauge in the sense of other gauge choices, but a unique representation of a gauge theory with redefined fields which do not transform under gauge transformations. In our gauge-free approach for the case of scalar QED, the action has an apparent similarity to the one obtained by Dolan-Jackiw by employing the so-called unitarity gauge. However, there is a crucial difference between their approach and ours in the functional integral. We include the physical constraint on the vector potential $\partial \cdot \mathbf{A}_{\mathcal{P}} = 0$ in the functional integration which is absent in [9] (See section 3). In the same spirit, we have also shown earlier that the Higgs-weak vector boson sector of the standard electroweak theory can also be rewritten in terms of manifestly gauge inert degrees of freedom [7]. The functional quantization of scalar quantum electrodynamics leads at the quantum level to a one loop effective potential which realizes the Coleman-Weinberg mechanism of mass generation in a gauge-free framework, thus resolving the issue of its gauge dependence. However, since the reparameterization invariance can only be ensured by treating the theory in the DeWitt-Vilkovisky (DV) approach, we have calculated the gauge-free theory according to the DV approach and get a different result from the one calculated earlier by Kunstatter [10]. This difference is an indication that by eschewing redundant field degrees of freedom from the outset, it is possible to obtain a unique result for the vector potential.

We may mention that there have been many efforts in the past towards identifying gauge invariant variables and formulating gauge theories in terms of these. See e.g. the recent paper by Ilderton et. al. [11] which provides a definitive guide to the literature of the mid-1990s on these efforts, including the authoritative contribution of Lavelle and McMullan [12]. Related to this earlier work, recently Niemi et. al. [13] and Faddeev [14] have proposed a gauge invariant description of the Higgs-gauge sector of standard electroweak theory whereby the Higgs field is given a novel interpretation as the dilaton in a *conformal* curved background. However, in

a completely gauge-free framework it has been shown that the novel interpretation of Niemi et. al. doesn't survive under quantization. Genuine one-loop effect actually cancels out the contribution of the dilaton field [7].

Although similar in spirit to some of these assays in a broad sense, we reiterate that our approach is distinct in that it is formulated in terms of a *local, physical* vector potential (instead of field strengths) as a *fundamental field variable*. In other words, we propose an alternative action/field equations as a new starting point rather than attempt to express the standard gauge theory action in terms of new variables.

The paper has been organised as follows: In the next section a brief review on different approaches to calculating the effective potential is given and the relative advantages of the gauge-free approach is discussed. In Section 3 we motivate our gauge-free approach by describing the functional quantization of vacuum electrodynamics. Then in Section 4, we deal with charged scalar fields (as already mentioned) and study the abelian Higgs mechanism. In Section 5 we consider the Coleman-Weinberg perturbative mass generation mechanism and obtain a gauge ambiguity free mass spectrum. Section 6 is devoted to a brief description of the DV method and a unique one loop effective-potential is derived adopting a combination of these two approaches. In Section 7 we generalize our gauge free approach to symmetric and anti-symmetric second rank tensor fields. We conclude in section 8 with a brief discussion on Yang Mills fields without Higgs scalars.

2 Effective Potential from different approaches

The dependence of effective potential on choice of gauge was first shown by Jackiw [2] in the context of scalar-QED. The one loop effective potential for scalar QED obtained by Jackiw in a general gauge $-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$ is gauge dependent:

$$V_{eff}(\phi_c) = \frac{\phi_c^4}{4!} \left[\lambda_R + \frac{\hbar}{8\pi^2} \left(\frac{5}{6} \lambda_R^2 + 9e^4 - \alpha e^2 \lambda_R \right) \ln \phi_c^2 \right] \quad (1)$$

It is thus possible to gauge away the one loop contribution to the effective potential by choosing α :

$$\alpha = \frac{5\lambda_R}{6e^2} + \frac{9e^2}{\lambda_R}$$

This is a serious problem because this raises the question of physicality of the effective potential itself. Soon after this result, Dolan and Jackiw [9] calculated the same effective potential in the so-called unitarity gauge and asserted that the theory in this gauge has no unphysical degrees of freedom, and hence the effective potential is *physical*. It is given by

$$V_U = \frac{1}{2} dm^2 \left(1 - \frac{\hbar\lambda}{64\pi^2} \right) \rho_c^2 + \frac{\lambda}{4!} \rho_c^4 \quad (2)$$

$$+ \frac{\hbar\lambda}{64\pi^2} \left[3e^4 \rho_c^4 \ln\left(\frac{\rho_c^2}{m^2}\right) + (m^2 + \lambda/2\rho_c^2)^2 \left(\ln\left(1 + \frac{\lambda\rho_c^2}{2m^2}\right) \right) \right], \quad (3)$$

and is clearly different from (1).

The problem of a non-unique one-loop effective potential had been discussed in several papers [10], [21]. It has been shown that the one loop effective potential depends not only upon the choice of gauge but also on the reparameterization of the fields. The reparameterization invariance means that the effective action coincides with the original one under a field redefinition $\phi \rightarrow \phi'$. DeWitt-Vilkovisky [4, 22] have introduced an effective action formalism which addresses both the problems (dependence of effective action on gauge fixing condition and on reparameterization of the fields) and provides a reasonable solution ! A lot of work have been done on the issue of gauge and parameterization dependence of the effective potential and notable amongst those is the result obtained by Kunstatter [10] in DV approach.

$$V_{eff}(\rho_c) = \frac{\lambda}{4!}\rho_c^4 + \frac{\hbar}{64\pi^2} \left(3e^4 + \frac{5}{18}\lambda^2 + \frac{2}{3}\lambda e^2 \right) \rho_c^4 \left[\log \frac{\rho_c^2}{M^2} - \frac{25}{6} \right]. \quad (4)$$

However, DeWitt-Vilkovisky's method does indeed need gauge fixing of the fluctuating gauge degrees of freedom which are being integrated over in the partition function. This of course is easily obviated by the use of the gauge free approach adopted in the present paper. Thus, the calculation of the effective potential becomes much easier in the gauge-free DV approach since the unphysical electrodynamic degrees of freedom are absent from the outset, and all fields are manifestly inert under $U(1)$ gauge transformations. Later we shall explicitly calculate the one-loop effective potential for scalar QED in the gauge-free DV approach and show that the potential does get modified from our earlier result and it also differs from the one obtained by Kunstatter [10].

3 Gauge Free Vacuum electrodynamics

We start with the Maxwell Electrodynamics to develop the idea of gauge-free quantization. For the standard gauge potential (abelian) one form A and semi-infinite curve C from spatial infinity to x ,

$$\mathbf{A}_C(x) \equiv h_{C(\infty,x)}[A](A + d)(h_{C(\infty,x)}[A])^{-1} \quad (5)$$

$$= A - d \int_{C(\infty,x)} A \quad (6)$$

For another semi-infinite oriented curve C' from ∞ to the point, it is easy to see that

$$\mathbf{A}_C - \mathbf{A}_{C'} = d \left[\int_{C'} - \int_C \right] A \quad (7)$$

$$= d \int_{S_{CC'}} dA \quad (8)$$

where, the second line follows from the first by Stokes theorem with $S_{CC'}$ being the surface bounded by the two semi-infinite curves C and C' from spatial infinity to x . The right hand side of the second equation vanishes because $d^2 = 0$. In other words, even though A_C formally depends on the curve C and is expected to be non-local, actually it is independent of C and

hence local. This also agrees with the fact the A_C is gauge-independent $\forall C$. Hence we drop the subscript C in what follows. Now, observe that

$$\mathbf{A} = A - d \int_C A = A - d \int d^4 x' dy_a A^a(x') \delta^4(y - x') \quad (9)$$

Define the d'Alembert Green function $G(x, x')$ as $\square G(x, x') = \delta^4(x - x')$. This is consistent with

$$\partial_a G(x, x') = \int_{C(\infty, x)} dy_a \delta^4(x - y) \quad (10)$$

as can be seen by taking partial derivatives on both sides. Substituting eq (6) in (5), and performing a partial integration, we get

$$\mathbf{A} = A - d \int d^4 x' G(x, x') \partial' A(x') = \mathbf{A}_{\mathcal{P}} \quad (11)$$

Here $\mathbf{A}_{\mathcal{P}}$ denotes a spacetime transverse physical vector which can easily be proved as follows.

$$\begin{aligned} \mathbf{A}_{\mathcal{P}} &= A - d \int d^4 x' G(x, x') \partial' A(x') \\ \partial \cdot \mathbf{A}_{\mathcal{P}} &= \partial \cdot A - \square \int d^4 x' \partial' A(x') G(x - x') \\ &= 0 \end{aligned} \quad (12)$$

With this physical field $\mathbf{A}_{\mathcal{P}}$ we now write an action for Maxwell Electrodynamics.

$$\partial \cdot \mathbf{A}_{\mathcal{P}} = 0 \quad (13)$$

We emphasize the fact that no gauge fixing needs to be employed here; but since the functional integral describing the vacuum-to-vacuum amplitude is over all configurations of the vector field $\mathbf{A}_{\mathcal{P}}$, the transversality constraint must be directly inserted into the integral to ensure that the integral is only over transverse field configurations. The gauge-free formulation for vacuum electrodynamics starts with the action

$$\begin{aligned} S[\mathbf{A}_{\mathcal{P}}, \Lambda; \tilde{\mathbf{J}}] &= \int \left[-\frac{1}{2} \partial_\mu A_{\mathcal{P}\nu} \partial^\mu A_{\mathcal{P}}^\nu + \tilde{J} \cdot A_{\mathcal{P}} + \Lambda \partial \cdot A_{\mathcal{P}} \right] \\ &= \int \left[-\frac{1}{2} \partial_\mu A_{\mathcal{P}\nu} \partial^\mu A_{\mathcal{P}}^\nu + J \cdot A_{\mathcal{P}} \right]. \end{aligned} \quad (14)$$

The second line above follows from the first by eliminating the Lagrange multiplier field Λ through its equation of motion and defining J_μ such that $\partial \cdot J = 0$. The relevant vacuum-to-vacuum amplitude (in presence of a transverse source) is given by

$$\begin{aligned} Z[\mathbf{J}] &= \int \mathcal{D}\mathbf{A}_{\mathcal{P}} \exp i \left(\frac{1}{2} A_{\mathcal{P}}^\mu \square A_{\mathcal{P}\mu} + \int d^4 x \mathbf{J} \cdot \mathbf{A}_{\mathcal{P}} \right) \delta[\partial_\mu \mathbf{A}_{\mathcal{P}}^\mu] \\ &= \int \mathcal{D}\mathcal{S} \mathcal{D}\mathbf{A}_{\mathcal{P}} \exp i \int d^4 x \left[\frac{1}{2} A_{\mathcal{P}}^\mu \square A_{\mathcal{P}\mu} + (J_\mu - \partial_\mu \mathcal{S}) A_{\mathcal{P}}^\mu \right]. \end{aligned} \quad (15)$$

In the second line of (15) we have introduced an auxiliary scalar field \mathcal{S} which acts as the Lagrange multiplier for the physical constraint (13). After integrating over $\mathbf{A}_{\mathcal{P}}$ and auxiliary field we get,

$$Z[\mathbf{J}] = N \exp \frac{-i}{2} \left[J^\mu \left(\frac{\eta_{\mu\nu}}{k^2} - \frac{k_\mu k_\nu}{k^4} \right) J^\nu \right] \quad (16)$$

A series of partial integrations and using the transversality of the current density \mathbf{J} , and also identities like $\partial_x \mathcal{G}(x-y) = -\partial_y \mathcal{D}(x-y)$ leads to this simple expression

It is now straightforward to extract the free photon propagator from eq. (16):

$$\mathcal{D}_{\mu\nu}(k) \equiv \frac{1}{2} \frac{\delta^2 W[\mathbf{J}]}{\delta J^\mu(k) \delta J^\nu(-k)} \Big|_{\mathbf{J}=0} \quad (17)$$

$$= \frac{1}{k^2 + i\epsilon} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 + i\epsilon} \right) \quad (18)$$

Clearly, this propagator does not possess any gauge artifacts. Where we have introduced the generating functional for connected Green's function via $W[\mathbf{J}] = -i \text{Log} Z[\mathbf{J}]$

In our gauge-free formulation, spacetime divergencelessness is *not* a matter of choice, it is a defining feature of what we mean by electromagnetism. Finally, note also that the free photon propagator falls off as $1/k^2$ for large momentum, as is expected for a *local* field.

4 Gauge-free electrodynamics with sources

4.1 Charged matter fields

All charged matter fields are complex fields Φ such that they can be ‘radially’ decomposed : $\Phi = \phi \exp i\theta$ where ϕ carries all the spin degrees of freedom of Φ and the phase field θ is a scalar field which appears in the action only through its first order derivative $\partial\theta$: $S[\Phi] = S[\phi, \partial\theta]$. The gauge-free prescription for coupling the gauge-free vector potential $\mathbf{A}_{\mathcal{P}}$ to Φ is exceedingly simple : leaving ϕ as it is in the action, simply replace $\partial\theta \rightarrow \partial\theta - eA_{\mathcal{P}}$, so that $S[\Phi] \rightarrow S[\phi, \partial\theta - eA_{\mathcal{P}}] + S_{free}[A_{\mathcal{P}}]$. Recall of course that the gauge-free $\mathbf{A}_{\mathcal{P}}$ is subject to the 4-divergencelessness constraint (13). The interaction with matter for this vector potential is merely to add a *physical longitudinal* part to it so that potentially it can now turn massive even in the weak coupling limit, depending upon the form of $S[\Phi]$. An example of this is the Abelian Higgs model of scalar electrodynamics [15].

4.2 Abelian Higgs Model

A charged scalar admits the radial decomposition $\phi = (\rho/\sqrt{2}) \exp i\theta$ where ρ and θ are both to be treated as physical fields. With this decomposition, the action of the complex scalar field appears as (suppressing obvious indices)

$$S_0[\rho, \theta] = \int d^4x \left[\frac{1}{2} (\partial\rho)^2 + \frac{1}{2} \rho^2 (\partial\theta)^2 - V(\rho) \right] . \quad (19)$$

This action (19) is invariant under the global $U(1)$ transformations $\rho \rightarrow \rho$, $\theta \rightarrow \theta + \omega$ where ω is a real constant.

Observe now that one can define $\Theta \equiv \theta - ea$ where a is introduced as part of the standard $U(1)$ gauge potential which carries the entire gauge transformation $a \rightarrow a + e^{-1}\omega$ when one couples the scalar theory to the standard gauge field A_μ . It is obvious that Θ is invariant under gauge transformations. Following our prescription above, coupling to the physical electromagnetic vector potential is obtained through the action (dropping obvious indices)

$$S[\rho, \Theta, \mathbf{A}_\mathcal{P}] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(\mathbf{A}_\mathcal{P} - e^{-1}\partial\Theta)^2 - \frac{1}{2}(\partial A_\mathcal{P})^2 - V(\rho) \right] \quad (20)$$

where $V(\rho)$ is the scalar potential, and $\mathbf{A}_\mathcal{P}$ obeys the divergenceless constraint (13). It is interesting that the phase field Θ occurs in the action only through the combination $\mathbf{A}_\mathcal{P} - e^{-1}\partial\Theta$; this implies that the shift $\Theta \rightarrow \Theta + \text{const.}$ is still a symmetry of the action. However, since there is no canonical kinetic energy term for Θ , it is hard to associate a propagating degree of freedom with Θ . Indeed, if one first makes a field redefinition

$$Y_\mu \equiv A_{\mathcal{P}\mu} - e^{-1} \partial_\mu \Theta. \quad (21)$$

the Θ can be completely absorbed into the new vector field \mathbf{Y}_μ , appearing only in the constraint which replaces (13)

$$\partial \cdot \mathbf{Y} = -\square \Theta. \quad (22)$$

This implies that \mathbf{Y} has three *physical* polarizations rather than the two that $\mathbf{A}_\mathcal{P}$ had. However, this does not immediately imply that \mathbf{Y} has acquired a mass. Upon eliminating Θ through the constraint (22), eq. (20) assumes the form

$$S[\rho, \mathbf{Y}] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}Y^a ((\square + e^2\rho^2)\eta_{ab} - \partial_a\partial_b) Y^b - V(\rho) \right], \quad (23)$$

This is the gauge-free Abelian Higgs model.

One can now think of two kinds of scalar potentials $V(\rho)$: one for which the minimum of the potential $\langle\rho\rangle = 0$ and the other for which the minimum lies away from the origin $\langle\rho\rangle = \rho_c \neq 0$. It is this second case which is of interest to us. If $V(\rho)$ has a minimum at $\rho = \rho_c \neq 0$ one now also defines $\rho \rightarrow \rho + \rho_c$, it is easy to see that the \mathbf{Y} acquires a mass $m_\mathbf{Y}^2 = e^2\rho_c^2$ while the ρ also acquires a mass $m_\rho^2 = V''(\rho_c)$. This is precisely the manner in which a *physical* longitudinal degree of freedom conjoins the photon field to produce a massive vector boson. In doing so, the new vector potential \mathbf{Y} is no longer subject to the transversality constraint (13). It thus has one degree of freedom more than the $\mathbf{A}_\mathcal{P}$. Observe that the Higgs phenomenon of mass generation **did not involve any symmetry breaking at all**, reminding us of Elitzur's theorem [16] proved for QED on a cubic lattice. The vacuum expectation value $\rho_c \equiv \langle\rho\rangle$ does not break any continuous symmetry at all. *The Higgs mechanism is a gauge-free mechanism of mass generation, involving neither symmetry breaking of any sort, nor unphysical particles in the spectrum.*

Before closing this subsection, we point out that this aspect of the phase field attaching itself to the photon field as a *physical* longitudinal piece, is not confined to charged scalar fields. Consider for instance a free charged Dirac field given by the action

$$S[\psi] = \int d^4x \bar{\psi}(i\gamma \cdot \partial - m)\psi . \quad (24)$$

Performing the ‘radial decomposition’ $\psi = \chi \exp i\theta$ this reduces to

$$S[\chi, \theta] = \int d^4x (\bar{\chi}(i\gamma \cdot \partial - m)\chi - \bar{\chi}\gamma \cdot \partial\theta\chi) . \quad (25)$$

This action is of course invariant under the global $U(1)$ transformations $\chi \rightarrow \chi$, $\theta \rightarrow \theta + \omega$ for a constant ω . Employing our prescription above for coupling this field to the physical electromagnetic vector potential, we notice that the action now reads

$$S[\chi, \theta] = \int d^4x (\bar{\chi}(i\gamma \cdot \partial - m)\chi - \bar{\chi}\gamma\chi \cdot (\partial\theta - eA\mathcal{P})) . \quad (26)$$

It is obvious from the above that under any interaction, the vector potential is *poised* to pick up a physical longitudinal piece ($\partial\theta$) corresponding to the ‘charge mode’. However, in this case there is no mechanism (at tree level) of mass generation due to the absence of a ‘seagull’ term. But this could be an artifact of weak coupling. In the 1+1 dimensional quantum electrodynamics model analyzed half a century ago by Schwinger [17], the photon field does pick up a manifestly gauge invariant mass as an exact dynamical result.

5 Gauge-free scalar QED: Coleman-Weinberg Mechanism

The Coleman-Weinberg mechanism [1] is a radiative mechanism whereby a scalar electrodynamics theory with massless photons and charged scalar bosons, changes its spectrum due to perturbative quantum corrections. Both the neutral component of the scalar boson and the vector boson acquire physical masses given by the parameters of the theory. In its incipient formulation, the mechanism has been shown to be gauge-dependent [2], thereby casting doubt on its physicality. Using the gauge free reformulation given above, we compute in this section the one loop effective potential of the theory, and argue that the effect is physical at this level.

The action for the theory is already given above eq.(20), with the choice $V(\rho) = (\lambda/4!)\rho^4$. Following [1], the theory is quantized using the functional integral formalism. In the standard formulation of QED, one needs to resort to the Faddeev-Popov technique of gauge fixing and extracting the infinite volume factor associated with the group of gauge transformations, from the vacuum persistence amplitude (generating functional for all Green’s functions), in order that this amplitude does not diverge upon integrating over gauge equivalent copies of the gauge potential. In the gauge free approach here, this technique is not necessary. The integration over the transverse gauge potential is, of course, restricted to configurations that obey the spacetime transversality condition (13). Since the integration variables are unambiguous, the task, at least at the one loop level, is simpler.

The generating functional is thus given by

$$Z[J, J', \mathbf{J}] = e^{\frac{i}{\hbar} W[J, J', \mathbf{J}]} = \int \mathcal{D}\rho \, \mathcal{D}\Theta \, \mathcal{D}\mathbf{A}_{\mathcal{P}} \exp \frac{i}{\hbar} \left[S[\rho, \Theta, \mathbf{A}_{\mathcal{P}}] + \int d^4x (J\rho + J'\Theta + \mathbf{J} \cdot \mathbf{A}_{\mathcal{P}}) \right] \cdot \delta[\partial_\mu A_{\mathcal{P}}^\mu] . \quad (27)$$

Here, the integration measures $\mathcal{D}\rho = \Pi_x d\rho(x)$, $\mathcal{D}\mathbf{A}_{\mathcal{P}} = \Pi d\mathbf{A}_{\mathcal{P}}$, but the remaining measure $\mathcal{D}\Theta = \text{Det}\rho^2 \Pi_x d\Theta(x)$. The extra factor of $\text{Det}\rho^2$ can be seen to arise if one begins with the generating functional first expressed as functional integrals over a complex scalar field and its complex conjugate. Alternatively, one can obtain the configuration space functional integral starting with the functional integral over phase space. Integration over the momentum conjugate to Θ produces the same factor [18].

Indeed, it is a similar factor which has been interpreted in [14] as representative of a background spacetime which is *conformally* flat, rather than flat, with the ‘radial’ component of the Higgs field ρ playing the role of the conformal mode. In [13], a slightly different interpretation is given of this radial Higgs field as a *dilaton* field. Formally, there is indeed novelty in both interpretations. However, when perturbative effects are included, at least at the one loop level, such interpretations will be seen to be in need of modification to account for scaling violations due to renormalization [7].

The effective action $\Gamma[\Phi]$ which is the generating functional for one particle irreducible diagrams (1PI), is generically defined as usual through the Legendre transformation

$$\begin{aligned} \Gamma[\Phi] &= W[\mathcal{J}] - \int d^4x \, \mathcal{J} \cdot \Phi \\ \Phi &= \frac{\delta W[\mathcal{J}]}{\delta \mathcal{J}} , \end{aligned} \quad (28)$$

where, we have collectively labeled all background fields as Φ and the sources as \mathcal{J} , and $W[\mathcal{J}]$, we recall, is the generating functional of connected Green’s functions.

The Effective Action can be derived iteratively from the integro-differential equation

$$\exp(i\Gamma[\Phi]) = \int D\phi \exp \left(iS[\phi] + i \int d^4x \, (\phi - \Phi) \frac{\delta \Gamma}{\delta \Phi} \right) \quad (29)$$

where we have used the equation of motion for the effective action

$$\frac{\delta \Gamma[\Phi]}{\delta \Phi} = -\mathcal{J}(x, \Phi) \quad (30)$$

The r.h.s. of this equation is a function of configuration space variable x but at the same time a functional of field Φ . The task is to compute $\Gamma[\Phi]$ to $\mathcal{O}(\hbar)$ with a view to eventually obtaining the one loop effective potential defined by the relation

$$V_{eff}(\phi_0) \equiv - \Gamma(\Phi)|_{\Phi=\phi_0} \left(\int d^4x \right)^{-1} , \quad (31)$$

where, ϕ_0 are spacetime independent. Observe that $V_{eff}(\phi_0)$ is the generating functional for 1PI graphs with vanishing external momenta. Even though the scalar potential is classically

scale invariant, a mass scale is generated through renormalization in the quantum theory, which breaks this scale invariance. The effective potential may thus have a minimum away from the origin in ρ -space, defined in terms of the renormalization mass scale, which, in turn, relates to values of the dimensionless physical parameters of the theory (dimensional *transmutation* [1]).

Instead of evaluation of the functional integral over the Θ and $\mathbf{A}_{\mathcal{P}}$ fields, we make a change of basis to Θ and \mathbf{Y} via (21) and make use of the action (23) which is independent of Θ . The latter appears only in the constraint which now becomes a statement of non-transversality in spacetime of the \mathbf{Y} field. Θ can be simply integrated out, leaving behind a field-independent normalization which we set to unity. The integration over ρ involves a saddle-point approximation around a field ρ_c which may be called a ‘quantum’ field, since it is the solution of the classical ρ -equation of motion augmented by $\mathcal{O}(\hbar)$ corrections. With no gauge ambiguities anywhere, there is no question of gauge fixing; functional integration over the physical vector potential \mathbf{Y} can be performed straightforwardly.

Following ref. [2], the one loop effective action is given schematically by

$$\Gamma^{(1)}[\rho_c] = S[\rho_c, 0, 0] - i\hbar Z^{(1)}[\rho_c], \quad (32)$$

where,

$$Z^{(1)}[\rho_c] \equiv \int \mathcal{D}\rho \mathcal{D}\mathbf{Y} \exp \frac{i}{2\hbar} \left[\int d^4x d^4y \rho(x) \mathcal{M}_{\rho\rho}(x, y) \rho(y) + Y^\mu(x) \mathcal{M}_{Y_\mu Y_\nu}(x, y) Y^\nu(y) \right], \quad (33)$$

with, generically,

$$\mathcal{M}_{AB}(x, y) \equiv \left(\frac{\delta^2 S[\Phi]}{\delta \Phi_A(x) \delta \Phi_B(y)} \right)_{\Phi=\rho_c, 0, 0}. \quad (34)$$

Since our object of interest is the one loop effective potential, we restrict ourselves to a saddle point ρ_c which is spacetime independent. The matrices \mathcal{M} turn out to be diagonal in field space for the purpose of a one loop computation, with entries

$$\begin{aligned} \mathcal{M}_{\rho\rho} &= - \left(\square + \frac{\lambda}{2} \rho_c^2 \right) \delta^{(4)}(x - y) \\ \mathcal{M}_{Y_\mu Y_\nu} &= [\eta_{\mu\nu} (\square + e^2 \rho_c^2) - \partial_\mu \partial_\nu] \delta^{(4)}(x - y). \end{aligned} \quad (35)$$

One obtains easily

$$Z^{(1)}[\rho_c] = (\text{Det} [\mathcal{M}_{\rho\rho} \mathcal{M}_{YY}])^{-1/2}, \quad (36)$$

The functional determinants are evaluated in momentum space following [2], and one obtains for the one loop effective potential, using eq. (31), the expression

$$\begin{aligned} V_{eff}(\rho_c) &= \frac{1}{4!} \lambda \rho_c^4 + i\hbar \int d^4k \log \left[(-k^2 + e^2 \rho_c^2)^{3/2} (-k^2 + \lambda \rho_c^2)^{1/2} \right] \\ &+ \frac{1}{2} B \rho_c^2 + \frac{1}{4!} C \rho_c^4, \end{aligned} \quad (37)$$

where B and C are respectively the mass and coupling constant counterterms. The momentum integral is performed with a Lorentz-invariant cut-off $k^2 = \Lambda^2$, yielding

$$\begin{aligned}
V_{eff}(\rho_c) &= \frac{1}{4!}\rho_c^4 + \frac{1}{2}B\rho_c^2 + \frac{1}{4!}C\rho_c^4 \\
&+ \frac{\hbar\rho_c^2\Lambda^2}{32\pi^2}\left(\frac{1}{2}\lambda + 3e^2\right) \\
&+ \frac{\hbar\rho_c^4}{64\pi^2}\left[\frac{1}{4}\lambda^2\left(\log\frac{\lambda\rho_c^2}{2\Lambda^2} - \frac{1}{2}\right) + 3e^4\left(\log\frac{e^2\rho_c^2}{\Lambda^2} - \frac{1}{2}\right)\right]
\end{aligned} \tag{38}$$

We remark here that in these manipulations, a $\exp(-\log\rho^2)$ term is generated in the one loop partition function, which cancels *exactly* against an identical term $Det\rho^2$ arising in the formal measure as discussed after eq. (27). This is precisely the point that was made earlier: the interpretation of that extra local factor in the formal functional measure as some sort of conformal mode in a conformally flat background is subject to some modification at the one loop level, since that factor is *eliminated* by a one loop contribution to the partition function. This has been anticipated in ref. [19] where an attempt has been made to give an alternate interpretation in terms of a ‘gauge-dependent gravity’. Perhaps one can use compensator fields to account for this loss of scale invariance due to renormalization effects, in order to resurrect the novel interpretation proposed in [13], [14].

The mass and coupling constant renormalizations B and C are fixed through the renormalization conditions

$$\left.\frac{d^2V}{d\rho_c^2}\right|_{\rho_c=M} = 0 \tag{39}$$

$$\left.\frac{d^4V}{d\rho_c^4}\right|_{\rho_c=M} = \lambda \tag{40}$$

leading to the renormalized one loop effective potential

$$\begin{aligned}
V_{eff}(\rho_c) &= \frac{\lambda}{4!}\rho_c^4 + \rho_c^2 M^2 \left[-\frac{\lambda}{4} + \frac{9}{32\pi^2}(3e^4 + \frac{1}{2}\lambda^2)\right] \\
&+ \left(\frac{3e^4}{64\pi^2} + \frac{\lambda^2}{256\pi^2}\right)\hbar\rho_c^4 \left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right].
\end{aligned} \tag{41}$$

In the usual approach, with or without the presence of θ (The latter case is usually regarded as “unitarity gauge”) as a dynamical variable in the action of scalar-QED the expression of one-loop effective potential does not agree [9] with each other. This is not surprising to have different results for the two cases. If we carefully look into these two theories then we can find that in the latter, there the vertex $\rho^2 \partial_\mu \theta A^\mu$ is absent. Therefore, it is obvious that the one-loop calculation starting from these two actions lead us to different results. However, in gauge-free approach the calculation from two actions (20) and (23) yields same result.

The potential has an extremum at $\rho_c = \langle \rho \rangle(M)$ leading eventually to the ratio of the squared masses of the Higgs boson to the photon

$$\frac{m_H^2}{m_A^2} = \frac{1}{e^2} \left[\frac{1}{3} \lambda - \left(\frac{3e^4}{8\pi^2} + \frac{\lambda^2}{32\pi^2} \right) \log \frac{\langle \rho \rangle^2}{M^2} - \frac{e^4}{\pi^2} - \frac{\lambda^2}{12\pi^2} \right] \quad (42)$$

The derivation of the mass ratio of the Higgs mass to the photon seemingly went through without any gauge fixing, since all fields being functionally integrated over are *physical* fields without any gauge ambiguity. The result (42) is thus a ‘physical’ result in this toy model where the photon acquires a mass. Notice that unlike in the original Coleman-Weinberg paper, we did not make an approximation of choosing $\lambda \sim e^4$, to drop terms of $O(\lambda^2)$. Thus, even though our result agrees with the earlier papers qualitatively, there are significant quantitative differences. However, the point in this section is not so much the result of the computation of the mass ratio, but the observation that the effect is physical and not a gauge artifact.

6 DeWitt-Vilkovisky Effective Action

We give a brief outline of the Vilkovisky’s unique effective action here. It was pointed out by DeWitt that we must evaluate the effective action more carefully by treating the space of field configurations as a manifold. The problem with conventional formalism of effective action lies in the fact that the eqn. (29)

$$\exp(i\Gamma[\Phi]) = \int D\phi \exp \left(iS[\phi] + i \int d^4x (\phi - \Phi) \frac{\delta\Gamma}{\delta\Phi} \right)$$

does not have a correct geometrical interpretation because the difference of two points in the configuration space \mathcal{M} , namely $\phi - \Phi$, in general, is not a vector on that space. This spoils the covariance of the expression $(\phi - \Phi) \frac{\delta\Gamma}{\delta\Phi}$ under field reparameterization s . Thus the effective action fails to be a scalar function on the configuration space \mathcal{M} . To get rid of this problem Vilkovisky proposed that $\phi - \Phi$ should be replaced by a two-point function $\sigma^i(\Phi, \phi)$ [we adopt DeWitt’s condensed notation here], which is a vector tangent to the curve joining the points Φ and ϕ . It is a vector with respect to the point Φ and a scalar w.r.t. the point ϕ . The properties of this bi-vector is discussed in detail in [20]. With this definition of $\sigma^i(\Phi, \phi)$ the effective action now been derived from

$$\exp(i\Gamma[\Phi]) = \int D\phi d\mu[\phi] \exp \left(iS[\phi] + i \int d^4x \sigma^i(\Phi, \phi) \frac{\delta\Gamma}{\delta\Phi} \right) \quad (43)$$

Now the r.h.s. of eqn. (43) becomes a scalar function of ϕ and the functional integral is independent of reparameterization of ϕ . We can expand $\sigma^i(\Phi, \phi)$ in powers of $\Phi - \phi$ with co-efficients evaluated at Φ .

$$\sigma^i(\Phi, \phi) = (\Phi - \phi)^i - \frac{1}{2} \Gamma_{mn}^i(\Phi) (\phi - \Phi)^m (\phi - \Phi)^n + \dots \quad (44)$$

In one-loop approximations one gets

$$\Gamma_{DV}^{(1)}[\Phi] = S(\Phi) + \frac{\hbar}{i} \ln \mu[\phi] + \frac{\hbar}{2i} \text{Tr} \ln[\nabla_m \nabla_n S(\Phi)] + O(\hbar^2) \quad (45)$$

where ∇_m is the covariant derivative associated with connection Γ_{mn}^i . The connection is Christoffel and completely described by the metric on the manifold \mathcal{M} .

$$\Gamma_{mn}^i = \frac{1}{2} G^{ik} (G_{mk,n} + G_{nk,m} - G_{mn,k}) \quad (46)$$

and

$$\frac{\delta^2 S}{\delta \phi^m \delta \phi^n} \rightarrow \nabla_m \nabla_n S = \frac{\delta^2 S}{\delta \phi^m \delta \phi^n} - \Gamma_{mn}^i(\phi) \frac{\delta S}{\delta \phi^i}. \quad (47)$$

For gauge theories we must evaluate the effective action in physical configuration space i.e. the space of all the gauge fields modulo the possible gauge transformations. First consider the infinitesimal gauge transformation,

$$\delta \phi^i = K_\alpha^i[\phi] \epsilon^\alpha, \quad (48)$$

with $K_\alpha^i[\phi]$ being the generators of gauge transformation and ϵ^α the infinitesimal gauge-group parameters.

Let G_{ij} be the metric in the naive field space;

$$ds^2 = G_{ij} \delta \phi^i \delta \phi^j \quad (49)$$

The metric of the physical field space is given by,

$$ds_P^2 = \gamma_{ij} \delta \phi^i \delta \phi^j = G_{ij} \delta \phi_P^i \delta_P^j \quad (50)$$

where the physical field is defined as

$$\delta \phi_P^i = \Pi_j^i \delta \phi^j \quad (51)$$

The projector Π_j^i projects the vectors of naive field space onto a subspace of vectors which are perpendicular to the space of tangent vectors to the orbits generated by K_α^i .

$$\Pi_j^i = \delta_j^i - K_\alpha^i N^{\alpha\beta} K_\beta^l G_{lj} \quad (52)$$

with

$$N^{\alpha\beta} = \gamma^{\xi\alpha} \gamma^{\chi\beta} K_\xi^i K_\chi^j G_{ij} \quad (53)$$

The modification of the connection due to gauge field gives

$$\Gamma_{jk}^i = \Gamma_{jk}^i + T_{jk}^i \quad (54)$$

where we have ignored a piece proportional to K_α^i since it will annihilate the action and will not contribute to the one-loop order. The gauge-part of connection reads,

$$T_{jk}^i = K_{(i}^\alpha K_{j)}^\beta K_\alpha^l K_{\beta;l}^m - K_i^\alpha K_{\alpha;j}^m - K_j^\alpha K_{\alpha;i}^m \quad (55)$$

6.1 Scalar QED in Gauge-free DV approach

The detailed calculation of one-loop effective potential for scalar QED is given in [10]. We do not repeat it here but merely restate essential results of that work. The action for the scalar QED is

$$S[\rho, \theta, A] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(A - e^{-1}\partial\theta)^2 - \frac{1}{2}(\partial A)^2 - \frac{\lambda}{4!}\rho^4 \right], \quad (56)$$

The metrics in field space are given by

$$G_{\rho(x)\rho(y)} = \delta^4(x - y) \quad (57)$$

$$G_{\theta(x)\theta(y)} = \rho^2\delta^4(x - y) \quad (58)$$

$$G_{A_\mu(x)A_\nu(y)} = -\eta_{\mu\nu}\delta^4(x - y) \quad (59)$$

For scalar QED the correct measure which is invariant under general co-ordinate transformations in \mathcal{M} will contain the determinant of the metric. Thus here,

$$d\mu[\phi] = \sqrt{\det G} = \text{Det}|\rho(x)\delta^4(x - y)|$$

The only contribution to the one-loop effective potential from the Christoffel symbol Γ_{jk}^i is

$$\Gamma_{\theta(x)\theta(y)}^{\rho(z)} = -\frac{\lambda\rho_c^4}{6}\delta^4(x - y) \quad (60)$$

$$(61)$$

To get the contribution from gauge part of the connection T_{jk}^i we identify the generators of gauge transformations

$$K_y^{\rho(x)} = 0 \quad (62)$$

$$K_y^{\theta(x)} = e\delta^4(x - y) \quad (63)$$

$$K_y^{A_\mu(x)} = -\partial_\mu\delta^4(x - y) \quad (64)$$

with the partial derivative with respect to the first argument of the δ -function. We write down the non-trivial Γ_{jk}^i s. The calculation details can be found in [10].

$$T_{\theta(x)\theta(y)}^{\rho(z)} = e^2\rho_c^3[\delta^4(x - y)N^{zx} + \delta^4(z - x)N^{zy} - e^2\rho_c^2N^{xy}N^{xz}] \quad (65)$$

$$T_{A_\mu(x)A_\nu(y)}^{\rho(z)} = -e^4\rho_c^5\partial^\mu N^{yx}\partial^\nu N^{zx} \quad (66)$$

$$T_{A_\mu(x)\theta(y)}^{\rho(z)} = e\rho_c[\partial^\mu N^{xz}\delta^4(x - y) - e^2\rho_c^2\partial^\mu N^{xy}N^{xz}] \quad (67)$$

The one-loop effective potential calculated in this formalism turns out to be independent of the gauge parameter. In fact it is equal to the one calculated by Jackiw with $\alpha = -1$ [10].

$$V_{eff}(\rho_c) = \frac{\lambda}{4!}\rho_c^4 + \frac{\hbar}{64\pi^2} \left(3e^4 + \frac{5}{18}\lambda^2 + \frac{2}{3}\lambda e^2 \right) \rho_c^4 \left[\log \frac{\rho_c^2}{M^2} - \frac{25}{6} \right]. \quad (68)$$

Now, we turn to the case of gauge-free scalar QED. In our gauge-free approach the action from which we have calculated the effective potential is given by eqn. (20),

$$S[\rho, \Theta, \mathbf{A}_{\mathcal{P}}] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(\mathbf{A}_{\mathcal{P}} - e^{-1}\partial\Theta)^2 - \frac{1}{2}(\partial A_{\mathcal{P}})^2 - \frac{\lambda}{4!}\rho^4 \right],$$

Now for reparameterization invariance we apply DV technique to calculate the effective potential for this action. The metrics of the field space are

$$G_{\rho(x)\rho(y)} = \delta^4(x-y) \quad (69)$$

$$G_{\Theta(x)\Theta(y)} = \rho^2\delta^4(x-y) \quad (70)$$

$$G_{A_{\mu}(x)A_{\nu}(y)} = -\eta_{\mu\nu}\delta^4(x-y). \quad (71)$$

However, the only non-trivial contribution to the one-loop effective potential will be from Γ_{Θ}^{ρ} and an additional contribution occurs from our gauge-free conventional calculation.

$$\mathcal{M}_{\Theta(x)\Theta(y)} = -\rho_c^2 \left[-k^2 + \frac{\lambda\rho_c^2}{6} \right] \quad (72)$$

Since again the theory doesn't possess any non-vanishing gauge generators ($K_y^{\Theta(x)} = 0; K_y^{\mathbf{A}_{\mathcal{P}}(x)} = 0$) we don't have any gauge part of the connection. The one-loop effective potential in this gauge-free framework becomes:

$$\begin{aligned} V_{eff} = & \frac{\lambda\rho_c^4}{4!} - i\hbar \int \frac{d^4k}{(2\pi)^4} \log \rho_c + \frac{i\hbar}{2} \int \frac{d^4k}{(2\pi)^4} \log[-\rho_c^2 k^2] \\ & + \frac{i\hbar}{2} \int \frac{d^4k}{(2\pi)^4} \log(-k^2 + \frac{\lambda\rho_c^2}{6}) \\ & + i\hbar \int d^4k \log \left[(-k^2 + e^2\rho_c^2)^{4/2} (-k^2 + \lambda\rho_c^2)^{1/2} \right] \\ & + \frac{1}{2}i\hbar Tr \int d^4k \log \left[\frac{k^2}{k^2 - e^2\rho_c^2} \right] \end{aligned} \quad (73)$$

The first integral comes from the integration measure which exactly cancels a divergent part coming from the inverse propagator of Θ (the second integral). The last term is the contribution from the transversality constraint on $\mathbf{A}_{\mathcal{P}}$, as already included in eqn. (27). This result clearly differs from the earlier result calculated by Kunstatter in [10]. It also doesn't agree with the result obtained by gauge-free approach (38) due to the third integral in the r. h. s. of V_{eff} . This is due to the fact that we have ignored the reparameterization invariance of the gauge-theories which was not captured in the gauge-free non geometric approach. The renormalized one-loop effective potential becomes

$$V_{eff}(\rho_c) = \frac{\lambda}{4!}\rho_c^4 + \frac{1}{64\pi^2} \left(3e^4 + \frac{5\lambda^2}{18} \right) \hbar\rho_c^4 \left[\log \frac{\rho_c^2}{M^2} - \frac{25}{6} \right]. \quad (74)$$

This is the unique gauge-free Coleman-Weinberg potential for scalar QED and surprisingly this coincides with the result of Coleman-Weinberg's original paper! This is just a coincidence because this calculation doesn't involve any gauge fixing in oppose to the case of Coleman-Weinberg's paper. So this is indeed a unique result which is free of any background or fluctuation gauge ambiguities and also invariant under field reparameterization s.

7 Generalization

7.1 Kalb-Ramond two form potential

The Kalb-Ramond two form potential \mathbf{B} has a field strength $\mathbf{H} = d\mathbf{B}$ which is clearly invariant under the gauge transformation $\mathbf{B} \rightarrow \mathbf{B} + d\Lambda$ for any one form field Λ . Construct now the projected two form field $\mathbf{B}^T \equiv \mathcal{P} \otimes \mathcal{P}\mathbf{B}$. Since $\mathcal{P}df = 0 \ \forall f$, under the gauge transformation of \mathbf{B} , $\mathbf{B}^T \rightarrow \mathbf{B}^T + \mathcal{P} \otimes \mathcal{P}d\Lambda = \mathbf{B}^T$. Further, in a coordinate system,

$$\partial_\mu B^{T\mu\nu} = 0 \quad (75)$$

implying that it is indeed transverse. Finally, it is clear that $\mathbf{H} = d\mathbf{B} = d\mathbf{B}^T$, which means that \mathbf{B}^T is indeed the physical part of the two form potential.

As in the case of gauge free electrodynamics, one can formulate the theory of Kalb-Ramond fields purely in terms of a *physical* antisymmetric tensor potential $B_{\mu\nu}$ defined by the action

$$S_{KR} = \int d^4x \left(-\frac{1}{2} B_{\mathcal{P}\nu\rho} \square B_{\mathcal{P}}^{\nu\rho} + J_{\nu\rho} B_{\mathcal{P}}^{\nu\rho} \right), \quad (76)$$

where, $\partial^\mu B_{\mathcal{P}\mu\nu} = 0 = \partial^\mu J_{\mu\nu}$.

We once again ask how unique the potential $B_{\mathcal{P}\mu\nu}$ is. Observe that both the field equation and the divergenceless condition remain invariant under a gauge transformation $B_{\mathcal{P}\mu\nu} \rightarrow (B_{\mathcal{P}\mu\nu})^\Lambda = B_{\mathcal{P}\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]}$ where Λ_μ satisfies the equation $\square\Lambda_\mu - \partial_\mu\partial \cdot \Lambda = 0$. In contrast to the case of the graviton field, it is obvious that this equation has an infinity of *gauge equivalent* solutions, the equivalence being under $\Lambda_\mu \rightarrow \Lambda_\mu + \partial_\mu\omega$ for an arbitrary function ω . Restricting $\Lambda_\mu(\infty) = 0$ is not enough to make it vanish everywhere. We need to additionally restrict $\partial \cdot \Lambda = 0$ everywhere with the requirement that $\omega(\infty) = \text{const}$. This additional restriction appears necessary in this preliminary investigation to make the two form potential unique.

The reason why an identical procedure as for the photon or graviton field does not suffice to yield a gauge-free formulation of antisymmetric tensor potentials is because of the aspect of *reducibility* of these potentials: the vectorial gauge parameter of the two form potential itself has a gauge invariance. Perhaps our approach will need to be somewhat modified to produce a gauge-free theory of potentials that have a reducible gauge invariance.

8 Conclusion

Generalization of the foregoing approach to Yang Mills theories, as has already been mentioned, has been achieved in the context of the electroweak theory where Higgs scalars are assumed to be present [13, 14]. In these papers, a residual $U(1)$ gauge theory corresponding to the Maxwell

theory has been obtained. We have already been succeeded to wipe out the residual freedom from the theory and to rewrite it in terms of completely gauge-inert variables [7]. However, a comprehensive study of all quantum properties of such a formulation is under way and will be reported elsewhere.

For pure Yang Mills theories, the construction of a gauge-free alternative has not yet been attempted, even though lattice gauge theories represent an explicitly gauge invariant formulation. A local, gauge-free formulation of Yang Mills theories is not obviously in contradiction with extant ideas about colour confinement of quarks and gluons. This gives us the opportunity to attempt a construction of a physical *non-Abelian* one form in terms of the usual Yang-Mills gauge one form \mathbf{A} (which takes values in the Lie algebra of the gauge group \mathcal{G}).

Defining the holonomy along the curve C from y to x as $h_{C[y,x]}[\mathbf{A}] \equiv \mathbf{P} \exp \int_{C(y,x)} \mathbf{A}$, with \mathbf{P} denoting path ordering, we note that under local gauge transformations of the gauge potential $[\mathbf{A}(x)]^{\Omega(x)} = \Omega(x)^{-1}[\mathbf{A}(x) + d]\Omega(x)$, where $\Omega \in \mathcal{G}$ the holonomy variables transform as

$$h_{C(y,x)}[\mathbf{A}^\Omega] = \Omega^{-1}(y) h_{C(y,x)}[\mathbf{A}] \Omega(x) . \quad (77)$$

If we choose the point $y \rightarrow \infty$ and require $\Omega(\infty) = \mathcal{I}$, eqn. (77) now takes the form

$$h_{C(\infty,x)}[\mathbf{A}^\Omega] = h_{C(\infty,x)}[\mathbf{A}] \Omega(x) . \quad (78)$$

We now formally define a *local* one form potential $\mathcal{A}(x)$ as

$$\mathcal{A}(x) \equiv \int \mathcal{D}C \bar{\mathbf{A}}_{C(\infty,x)} \quad (79)$$

where,

$$\bar{\mathbf{A}}_{C(\infty,x)} \equiv h_{C(\infty,x)}[\mathbf{A}] (\mathbf{A} + d) (h_{C(\infty,x)}[\mathbf{A}])^{-1} . \quad (80)$$

The path integral symbol at this point is formal, and is meant to stand for some sort of averaging over all paths originating at asymptopia and extending upto the field point x . It is then easy to see that, under gauge transformations of \mathbf{A} and using eqn. (78),

$$\mathcal{A}^\Omega(x) = \mathcal{A}(x) . \quad (81)$$

What we have not been determined yet is what constraint replaces the divergencefree condition (13) for the Yang Mills one form \mathcal{A} , so that the physics of these local gauge-free one forms can be explored more thoroughly without gauge encumbrances. One also envisages application of these ideas to general relativity formulated as a gauge theory of Lorentz (or Poincaré) connection. We hope to discuss these and consequent issues elsewhere.

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